On Deciding Truth and Falsehood in Hyperbolic Digital Spaces[1]

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Abstract

In questo saggio approfondiamo il panorama complesso degli spazi digitali e le formidabili sfide che pongono alle euristiche umane. La peculiare struttura "iperbolica" che caratterizza questi spazi, in cui l'interazione reciproca fra le connessioni e le relazioni tra entità digitali li rende allo stesso tempo ricchi ed elusivi, serve come quadro fondamentale per la nostra successiva analisi, in cui ci concentriamo specificamente sul ruolo indispensabile svolto dagli algoritmi nel rendere questi spazi digitali navigabili.
Al centro della nostra esplorazione si trova la tesi centrale che fonda la nostra prospettiva: gli algoritmi sono indispensabili per permettere una navigazione digitale ma intrinsecamente inclini a introdurre pregiudizi nel processo di ricerca. In particolare, l'applicazione di algoritmi completamente imparziali comprometterebbe l'utilità stessa degli spazi digitali. La nostra posizione sottolinea l'equilibrio delicato tra gli imperativi dell'esplorazione e le necessità di personalizzazione negli ambienti digitali.
Analizziamo quindi esplicitamente il collegamento fra la natura iperbolica degli spazi digitali e le sfide inerenti ai nostri sforzi nella ricerca di informazioni. In questo contesto, sottolineiamo come gli algoritmi per classificare la veridicità delle informazioni digitali siano sempre vincolati da teoremi matematici fondamentali.
Concludiamo osservando come gli algoritmi, pur servendo ad amplificare le nostre capacità, nel mondo digitale, non possano mai sostituire completamente le complesse sfumature del giudizio umano e delle considerazioni etiche. La nostra tesi sull'interazione dinamica tra la navigazione algoritmica ed i processi decisionali umani sottolinea l'imperativo di riconoscere e convivere con le limitazioni intrinseche degli algoritmi.

Parole chiave: spazi digitali iperbolici, bias algoritmic, realtà multiple, ricerca di informazioni, bolle epistemiche.

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In this essay, we delve into the intricate landscape of digital spaces and the formidable challenges they pose to human heuristics. We commence by analysing the unique "hyperbolic" structure characterizing these spaces, where the interplay of connections and relationships between digital entities is both abundant and elusive. This initial exposition serves as a foundational framework for our subsequent in-depth analysis, specifically focusing on the indispensable role played by algorithms in rendering these digital landscapes navigable.
At the heart of our exploration lies a central thesis that defines our perspective: algorithms are simultaneously essential for the facilitation of efficient digital exploration and inherently predisposed to introducing biases into the process. We argue that the pursuit of entirely unbiased algorithms would obstruct the very usability of digital spaces. This stance underscores the delicate equilibrium between the imperatives of exploration and the need for personalization in digital environments.
We then draw explicit connections between the hyperbolic nature of digital spaces and the challenges inherent in our information-foraging endeavours. In this context, we examine how algorithms to classify the veracity of digital information are always constrained by fundamental mathematical theorems.
We conclude by observing how, within the digital realm, algorithms serve to amplify our capabilities, but they can never fully supplant the intricate complexities of human judgment and the nuanced considerations of ethics. Our thesis centres on the dynamic interplay between algorithmic navigation and human decision-making, underscoring the imperative to coexist with and acknowledge the inherent limitations of algorithms.

Keywords: hyperbolic digital spaces, algorithmic bias, multiple realities, information foraging, epistemic bubbles.

1. *Hyperbolic Networks*

In the beginning there was the Internet, born for military purposes (control and communication). Then, the scientific community expanded it to be a means of storage and exchange of knowledge as well as a collaborative environment that allowed breaking down geographical distances. Yet, the greatest revolution was the introduction of *hypertext links*. The ability of an object to refer to other objects (through a hypertext link) increases exponentially the possibilities of creating relationships between objects; data begins to enrich itself with meaning based on the references it has, but at the same time the choice of references – i.e. their contextualization – can change the meaning of the individual data point. This was the time when the Internet became navigable, thanks to the introduction of web pages (the so-called "World Wide Web" or WWW) containing links to other pages (hyperlinks), allowing users to "jump" from one page's content to another. Hyperlinks create a network between the pages (see Fig. 1); thus, during online navigation, we "move" on a network that, as we will see, has extremely unique characteristics. Notice that the fact that the verbal form "move" is of common usage indicates that we are already implicitly describing the Internet as a space. However, it is not a space we are accustomed to.

In fact, the "navigable" networks we are historically accustomed to are transport networks. For example, the London Underground can be seen as a network that connects stations and allows us to travel from one to another. Notably, the maps of subways we use are schematized using dots representing stations (the "navigable" objects) connected by lines that indicate the presence of links between objects. Road networks that connect cities or networks that bring gas to urban and industrial areas are additional examples of "navigable" networks that we are familiar with. In any case, we are dealing with "planar" networks, that are immersed on a two-dimensional and (locally) Euclidean surface[2]. This means that the exploration of such networks is subject to constraints. In particular, if I double the distance I can travel, statistically I will quadruple the number of reachable locations. In general, in a two-dimensional space by exploring up to a distance **L**, I can reach a number of objects that grow as **L x L**. Note that the growth law of the number of objects is related to the dimension of the space I am exploring: if I were in three dimensions, it would grow as **L x L x L**; in four, as **L x L x L x L**, and so on[3]. Actually, since we are used to living and interacting on the surface of the earth, our natural environment is two-dimensional: no wonder the idea of a flat earth is the easiest to accept since it corresponds to our daily experiences, even if it is contradictory to scientific evidence. Thus, navigable networks of which we historically have concrete experience are two-dimensional objects, and our implicit heuristics will try to reduce whatever space we are travelling to the spaces we are used to.

In the WWW, like in a subway where we can go from one station to another, we find ourselves in a different space where we travel among the pages. However, in this case, we do not have the full map, but only an indication of which are the next "stations" (i.e. the other pages we can jump to). Therefore, we find ourselves in the situation of exploring a space having only local information; it is like starting to explore a city from the central station without having a map: unless we are on a serendipity trip, it is not the ideal strategy. This strategy becomes even more ineffective when we characterize the structure of the WWW in an objective way, i.e.

[2] A planar network is a type of network that can be represented in a two-dimensional space without any lines or connections intersecting. In simpler terms, it's a network that can be drawn on a flat surface (like a piece of paper) without any of the lines crossing over each other.

[3] In a two-dimensional space, when we explore a distance marked as "**L**", the number of objects we can reach grows as "**L x L**". To put it simply, if you imagine moving on a flat surface, the area you can cover expands as the square of the distance you travel. This means that in a two-dimensional space, if you double your distance (**L**), you can potentially access four times as many objects. If we were in three dimensions, the growth would be "**L x L x L**", and in four dimensions, it would be "**L x L x L x L**", and so on. So, the dimension of the space you're in significantly influences how quickly you can access more objects as you explore further.

by mathematics and, in particular, by Network Theory[4]. From the mathematical point of view, Networks (or Graphs) describe systems composed of elements with mutual relations; the nodes of the network represent such elements, while a link between two nodes represents an existing relation among the corresponding elements (see Fig. 2).

Since these kinds of networks are not physical spaces, the natural way of defining the distance between two nodes is in terms of the minimum number of links that one must cross to go from one node to the other. Thus, one can start measuring the characteristics of abstract networks like the *diameter*, i.e. the maximum distance among the nodes. Surprisingly, by doing so, we have discovered that a common characteristic of networks created by hyperlinks (be it the WWW or the network of friends on a social media) is to be "small world"[5], i.e. they have exceptionally small diameters of the order of few jumps; as an example, in 1999 the WWW was already composed of about a trillion pages, however, the farthest page could be reached by just 19 "clicks"[6]. This means that in 1999 I could reach one thousand different pages in less than 7 clicks, or one million pages in less than 12 clicks; therefore, when "moving" in these abstract spaces we find ourselves in a completely different situation from what we are used to. For instance, let's imagine what would happen if the links on WWW pages resembled a structure we are familiar with, like a square network, such as the streets of Manhattan (see Fig. 3). In such a scenario, it would take around than 22 clicks to reach any of the nearest one thousand pages[7]. However, it would require around 700 clicks to reach any of the nearest one million pages. With some patience, it might even take about 700,000 clicks to reach any of the one trillion pages. So, on one hand, the WWW is an extremely navigable network (I can go wherever I want in just a few clicks, while in a planar network, there are pages that I would never reach because I would get tired of navigating before), on the other hand, it is clear that without a map, I risk getting lost or – even worse – never finding what I'm looking for.

Technically, the navigational space of the WWW has the structure of a random network or, more precisely, of a family of random networks called scale-free networks[8]. One characteristic of random networks is to be spaces with a locally hyperbolic structure, meaning that the neighbourhood of an object has a number of neighbours that grows more rapidly than in any Euclidean space (recall that we live in a space that is locally Euclidean). This means that, when we start exploring the WWW, we are moving in an alien space, totally different from the almost two-

[4] G. Caldarelli, M. Catanzaro, *Networks: A very short introduction*, vol. 33, Oxford University Press, Oxford 2012.
[5] L.A.N. Amaral *et al.*, *Classes of small-world networks*, in «Proceedings of the national academy of sciences», 97, n. 21, 2000, pp. 11149-11152.
[6] R. Albert, H. Jeong, A.-L. Barabási, *Diameter of the world-wide web*, in «Nature», 401, n. 6749, 1999, pp. 130-131.
[7] The points ad a distance (i.e. number of hops) less than or equal to D on a Manhattan grid are approximatively **2 x D x D** (see Fig. 3).
[8] A.-L. Barabási, R. Albert, *Emergence of scaling in random networks*, in «Science», 286, n. 5439, 1999, pp. 509-512.

dimensional space in which we evolved. Thus, a hyperbolic space is a space for which we have no natural means or organs of orientation, a space in which relying on intuition to explore can lead to results opposite to those desired. To gain a visual understanding of the way neighbourhoods are structured within a hyperbolic space, we can draw inspiration from the artistic work of M.C. Escher[9] (see Fig. 4). Escher's often featured intricate, tessellating patterns and optical illusions that provided a glimpse into non-Euclidean geometries, including the hyperbolic space. By examining Escher's creations, we can appreciate the counterintuitive nature of a hyperbolic space, where the neighbourhoods of objects exhibit an exponentially growing number of neighbours as one moves away from a central point. Escher's artwork offers a tangible representation of how objects in such a space connect and interrelate, helping us grasp the peculiar characteristics of hyperbolic digital environments.

The WWW is an example where *the network is explicit*, meaning its nodes (the pages) *explicitly* contain links to other nodes. A similar case is Wikipedia, where one can move from a topic to related entries. Recently, most scientific publications have transitioned online, featuring a "hypertextual" bibliography that directly links to cited articles when they are also available online. Thus, starting from a paper, one can explore its "neighbourhood" by clicking on the links; any scholar even with minimal online experience knows that analysing the material that can be found within just two clicks is already enough to require days, weeks, or even months of reading.

On the other hand, business models of Internet platforms are mainly based on *implicit networks*. In an implicit network, the link is *built a posteriori* using the data that often only the platform can access and is based on some similarity measures among the nodes (see Fig. 5). Implicit networks allow online platforms to build up groups of *targetable* users, a practice that is the cornerstone of the exponential growth of their revenues[10]. On the same footing, implicit networks are the cornerstone of propaganda: just as the holy grail of marketing is *consumer segmentation* (i.e. dividing consumers into classes, for each of which the ideal product and marketing strategy are known), *voter profiling* is the philosopher's stone of politics: knowing what to say, how to say it, to whom and when. However, the fact that each of us usually belongs to several distinct implicit networks and the multiplicity of the marketing and propaganda sources possibly mitigates such issues. At the same time, the evidence for the existence of large *echo chambers*[11], i.e. isolated user groups where ideological positions and monolithic beliefs circulate and amplify, introduces a possible vulnerability in the very foundations of liberal Western democracies[12].

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[9] See <http://pi.math.cornell.edu>.
[10] A. Scala, M. Delmastro, *The explosive value of the networks*, in «Scientific Reports», 13, 2023, 1037.
[11] M. Del Vicario *et al.*, *The spreading of misinformation online*, in «Proceedings of the national academy of Sciences», 113, n. 3, 2016, pp. 554-559.
[12] C.R. Sunstein, *Democracy and filtering*, in «Communications of the ACM», 47, n. 12, 2004, pp. 57-59; G. Pondrano Altavilla, A. Scala, *Ripensare i fondamenti della liberaldemocrazia nell'era di internet*, in «MicroMega», 7, 2018, p. 12.

As we conclude our examination of the hyperbolic attributes characterizing digital spaces, it becomes evident that the multifaceted nature of these environments necessitates algorithmic assistance to facilitate efficient navigation. In the ensuing section, we delve into the critical role of algorithms within digital spaces and the intrinsic biases they introduce. Furthermore, we explore the algorithmic limitations that surface when managing the abundant digital information landscape.

2. *Algorithms & the Construction of Digital Reality*

In the digital space, both explicit and implicit networks are mostly large, hyperbolic and small-world. Thus, the main issue is how to navigate such networks without getting lost. To such an aim, we need algorithms that filter out what we can see reducing the possible exploration paths. In fact, even after the WWW, the main barrier to Internet usage from the average user was the lack of an efficient search engine; indeed, in 1999 Windrum and Swan were still writing that «The most frequently discussed search engine problem is the data glut generated by automated engines. These typically generate concordances on far more links than the user has time to process, with little or no indication as to the nature of the pages and, hence, their suitability»[13].

Thus, it is not a case that the real penetration of the Internet among the average Joe occurred only when search engine technologies started to work. The only way to achieve this goal is to strongly restrict the possible paths you can take by having the search engine *choose for you* the possible directions to explore, even if this choice has the milder appearance of an ordered list of possibilities (choices can be influenced both by the order in which they are presented or even in the form they are expressed[14]). A further problem is that, since algorithms cannot really understand what we intend, they have to tune their choices on the rate of satisfaction of the inquirer (i.e. on the fact that he interacted with one of the first proposed choices), with a classical feedback loop control that restricts its proposals to what we have already liked, killing innovation.

Likewise, as soon as social media grew to more than a few thousand users, similar issues obliged to introduce algorithms that selected what to present on users' timelines. Again, the only possible choice was to apply the same kind of feedback loop, leading to the birth of the concept of echo chambers[15] and epistemic bubbles[16].

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[13] W. Paul, P. Swann, *Networks, noise and web navigation: Sustaining Metcalfe's law through technological innovation*, in «Research Memorandum», 9, 1999, Maastricht University, Maastricht Economic Research Institute on Innovation and Technology (MERIT). See <https://ideas.repec.org>.

[14] D. Kahneman, A. Amos Tversky, *Choices, values, and frames*, in «American psychologist», 39, n. 4, 1984, p. 341.

[15] C.R. Sunstein, # *Republic: Divided democracy in the age of social media*, Princeton University Press, New York 2018.

[16] C.T. Nguyen, *Echo chambers and epistemic bubbles*, in «Episteme», 17, n. 2, 2020, pp. 141-161.

However, as usual, it is a "you can't have your cake and eat it too" situation: whatever algorithm would be used for searching or presenting content, either blinds us from a large part of the digital world or produces practically random – and thus probably useless and/or uninteresting – data. Moreover, at the moment to the best of the author's knowledge, there is no way out from the confirmation-bias-alike feedback loop[17] that seems to be the only one to ensure retrieving/presenting content that the vast majority of users are happy with.

Thus, we have a digital space where even contrasting information can be found and where, once we find a specific type of information, the digital algorithms will often lead us to encounter even more content of a similar nature. It is a place where the exponential number of exploration paths allow us to build up multiple coherent realities (i.e. the nodes we know about and that we usually access). Being outside of the physical world, there are no physical laws to respect: the proofs or the disproofs that the earth is flat are themselves electronic documents, so as long as we remain in the digital world they have the same right to be considered true. The point is whether we will be able to separate the narrative realities of the digital space from our understanding of the non-digital space.

3. Algorithmic Limits & Social Choice

Up to now, we have argued that, given the hyperbolic character of digital spaces, it is impossible to explore such spaces without "biased" algorithms that restrict the exploration to a non-exponential number of possible paths. However, beyond such an intrinsic limit and in the context of these unique digital spaces, there are other limits to what algorithms can achieve. These additional limitations are not only independent of the space's structure but also rooted in well-known impossibility theorems[18]. In the following, we will delve deeper into these algorithmic limits and their implications for navigating the complex digital landscape.

Large-scale quantitative studies support the hypothesis that in digital spaces human and algorithmic biases concur to the formation of echo chambers and epistemic bubbles that exacerbate social interactions by enhancing opinions'

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[17] R.S. Nickerson, *Confirmation Bias: A Ubiquitous Phenomenon in Many Guises*, in «Review of General Psychology», 2, n. 2, 1998, pp. 175-220.

[18] In particular, we will highlight that there are fundamental limitations to what algorithms can achieve in the context of digital spaces due to two GödelGodel's incompleteness theorem and Arrow's theorem. GödelGodel's incompleteness theorem shows that there are statements or propositions within a formal system (in this case, an algorithmic system) that cannot be proven true or false based on the axioms or atomic truths of that system. In other words, there are limits to what algorithms can deduce or decide. Arrow's theorem is related to social choice theory and deals with the aggregation of individual preferences to make a collective decision. It demonstrates that no ranking algorithm based on preferences can satisfy a set of seemingly reasonable criteria simultaneously making it challenging to achieve a "fair" ranking algorithm in a democratic context.

polarisation[19]. To address these issues, one proposed approach is to encourage diverse sources of information and opinions[20], and to deploy fact-checking services to help in identifying and correcting "false information"[21]; however, regardless of the problem of defining "false information", it has been shown that exposing users to information contrasting their beliefs can backfire and increase polarization and extreme views[22].

Algorithmic approaches developed to combat fake news and misinformation use machine learning and natural language processing techniques to automatically identify and classify false or misleading information. Most common approaches include content-based detection (identifying patterns and features that are common in fake news articles) or network-based detection (identifying suspicious accounts or sources by analysing the patterns of sharing and engagement with specific sources)[23]. It is thus clear that there is already a problem in defining what is "fake" or "false": in fact, such algorithms must rely on external sources such as fact-checking websites or expert opinions to "learn" which news articles or social media posts are "truthful" and, being not foolproof, are often be used in conjunction with other strategies such as human fact-checking and critical thinking skills[24]. While in 2017 the US Federal Communication Commission argued that "public interest algorithms" can aid in identifying and publicizing fake news posts and therefore be a valuable tool to protect consumers[25] and Germany was heading toward legislation that could possibly promote over-censoring[26], at least in EU there was the awareness that it is not always clear how to identify objectionable content[27].

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19 E. Bakshy, S. Messing, L.A. Adamic, *Exposure to ideologically diverse news and opinion on Facebook*, in «Science», 348, n. 6239, 2015, pp. 1130-1132; A. Bessi *et al.*, *Science vs conspiracy: Collective narratives in the age of misinformation*, in «PloS one», 10, n. 2, 2015, e0118093 <https://doi.org/10.1371/journal.pone.0118093>.

20 M.A. Baum, T. Groeling, *New media and the polarization of American political discourse*, in «Political Communication», 25, n. 4, 2008, pp. 345-365; T.J. Leeper, R. Slothuus, *Political parties, motivated reasoning, and public opinion formation*, in «Political Psychology», 35, 2014, pp. 129-156.

21 B. Nyhan, J. Reifler, *When corrections fail: The persistence of political misperceptions*, in «Political Behavior», 32, n. 2, 2010, pp. 303-330.

22 *Ibidem*; F. Zollo *et al.*, *Debunking in a world of tribes*, in «PloS one», 12, n. 7, 2017, e0181821 <https://doi.org/10.1371/journal.pone.0181821>; C.A. Bail *et al.*, *Exposure to opposing views on social media can increase political polarization*, in «Proceedings of the National Academy of Sciences», 115, n. 37, 2018, pp. 9216-9221.

23 K. Shu *et al.*, *Fake news detection on social media: A data mining perspective*, in «ACM SIGKDD explorations newsletter», 19, n. 1, 2017, pp. 22-36.

24 X. Zhou, R. Zafarani, *A survey of fake news: Fundamental theories, detection methods, and opportunities*, in «ACM Computing Surveys (CSUR)», 53, n. 5, 2020, pp. 1-40.

25 T. Wheeler, *Using 'Public Interest Algorithms' to Tackle the Problems Created by Social Media Algorithms*, in «Brookings TechTank», November 1, 2017 <https://www.brookings.edu/articles>.

26 C. Radsch, *Proposed German Legislation Threatens Broad Internet Censorship*, in «Committee to Protect Journalists», April 20, 2017 <https://cpj.org/2017/04>.

27 European Digital Rights, *Recommendations on the German Bill "Improving Law Enforcement on Social Networks"*, June 20, 2017 <https://edri.org/files/consultations>.

The main problem with the algorithmic approach is the expectations of the users and of the policymakers. Most people regard algorithms (and science in general) as sources of univocal, "true" results and/or fair judgments/evaluations. When scientists advertise algorithms that "detect lies and untruthful facts", they exacerbate such false expectations.

Such misconceptions contrast with several theoretical results that have established sharp limits to what algorithms can afford: in fact, algorithms cannot even deduce all the "true" consequences of a basic set of atomic truths (Gödel's incompleteness theorem[28] / Turing's undecidability theorem[29]) or uniquely identify the general interest of a community from individual preferences (Arrow's theorem[30]).

The results of Gödel, Turing, and Church, among others, pertain to the potential deductions that an algorithmic system can derive from a set of atomic truths or axioms. It's important to note that these atomic truths represent our foundational assumptions. Even in this simplified context, Gödel's incompleteness theorem tells us that certain "true" consequences of our axioms cannot be algorithmically proven. Similarly, Turing's undecidability theorem reveals that for certain propositions, an algorithmic system will run indefinitely without ever reaching a conclusive answer. In addition, Church's theorem on the Undecidability of the Calculus of First Order Predicates, established in 1936[31], implies that there is no mechanical algorithmic procedure capable of determining the truth functionality of any formula within the language of first-order predicate logic. These results underscore the inherent challenges and limitations of algorithms in both digital spaces and formal logic. It's worth noting that we can always design algorithms that halt; however, Gödel, Turing, and Church's theorems address algorithmic systems with sufficient power to express at least basic arithmetic, suggesting that demanding algorithms to provide universally "true" answers, based on a set of "ground truths", would necessitate sacrificing their ability to comprehend even elementary mathematics.

A further limitation of what algorithms can do is Arrow's theorem, a fundamental result in social choice theory, which shows that no ranking algorithm based on preferences can simultaneously satisfy a set of seemingly reasonable criteria. These criteria include:

1. Pareto efficiency: If everyone prefers one option to another, that should be the first in the rank.

2. Independence of irrelevant alternatives: The ranking of two options should not depend on the presence or absence of other, irrelevant options.

3. Non-dictatorship: No individual should be able to determine the group's preference on their own.

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[28] E. Nagel, J.R. Newman, *Gödel's Proof*, Routledge, New York 1958.

[29] A.M. Turing, *On computable numbers, with an application to the Entscheidungsproblem*, in «Proceedings of the London Mathematical Society», 58, 1936, pp. 230-265.

[30] A. Sen, *Collective Choice and Social Welfare*, Penguin Books, London 2017.

[31] A. Church, *A note on the Entscheidungsproblem*, in «The journal of symbolic logic», 1, n. 1, 1936, pp. 40-41.

This theorem has important implications for democratic decision-making (it is normally formulated in terms of a voting system) and has sparked much research into alternative methods for aggregating individual preferences. Therefore, it indicates that in the digital space, there is no way of bringing individuals to an agreement that is considered "fair" by everyone since, depending on the ranking algorithm, different groups would see their best choice as the preferred one. On the same line of thought, there is Sen's theorem on the liberal paradox[32], a mathematical proof that shows that it is impossible to construct a democratic voting system (aka a "democratic" ranking algorithm) that satisfies four reasonable criteria of fairness simultaneously. Like Arrow, Sen assumes non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives but adds the unrestricted domain criterion. The unrestricted domain means that any preference ordering of the individuals in the society can be used to determine the societal ordering of the alternatives. Sen's theorem shows that these four criteria are incompatible, and thus there is no "fair" ranking algorithm exists.

Thus, the core problem in applying algorithms to make decisions for us humans is the problem of algorithmic fairness[33]. The concept of fairness has been studied for centuries and is a fundamental principle of ethical conduct. However, it has a certain level of ambiguity to it, unlike the exact sciences. We use contextual criteria to assess whether an action is fair or not, which can vary and evolve over time. Fairness depends on the circumstances and takes into account factors such as power dynamics and historical injustices. These ongoing debates have led to discussions about what constitutes fairness in various situations, including in politics, economics, and social justice. While the ethical concept of fairness is characterized by productive ambiguity and variable standards, AI/ML (Artificial Intelligence/Machine Learning) algorithms operate through mathematical optimization methods. Their perspective is limited to the past and does not allow for interpretation against a meaningful world of future possibilities, unlike ethical thinking. Moreover, AI/ML produces answers that are ensured to be *statistically* reasonable only on their *training set*; in fact, to an AI/ML computation, there should always be attached scores for the expected accuracy of the answer and for the relevance of the question with respect to the training set.

Conclusions

Nowadays, we are embedded in digital spaces; however, their "hyperbolic" structure is not the space in which we have evolved; thus, our inborn heuristics can easily lead to making the wrong decision. Their structure is not intrinsically new: the connections

[32] A. Sen, *The impossibility of a Paretian liberal*, in «Journal of political economy», 78, n. 1, 1970, pp. 152-157.
[33] D. Pessach, E. Shmueli, *A review on fairness in machine learning*, in «ACM Computing Surveys (CSUR)», 55, n. 3, 2022, pp. 1-44.

shaping the digital space resemble the knowledge space of ideas and texts that scholars are used to navigating while doing research. However, it is not a coincidence that forming a scholar requires a long time: essentially, scholars learn to "move" in an abstract space totally different from the physical one we are born in. The main difference is that today everybody who has a smartphone is immersed, regardless of his formation, in such an abstract space.

The hyperbolic nature of the digital space on the one hand makes distances among its objects disappear, on the other hand makes it unfathomable since without guidance everybody is lost, even the most accomplished scholar. Given the humongous size of such space, only algorithms can set up for us paths to explore it fruitfully; however, setting these paths means that biases have been introduced. Therefore, asking for unbiased algorithms means destroying the usability of digital spaces. However, we could perhaps ask that such biases do not exacerbate our worst characteristics.

Finally, since we have to resort to algorithms to access and "live" in digital spaces, we must not forget what we cannot "ask" of algorithms. There are no algorithms that can tell us what is false or what is true. There are no algorithms that can solve discussions by making an indisputable choice respecting the preferences of a group of individuals. There are no absolutely "fair" algorithms, only statistically "fair" algorithms. Algorithms augment our capabilities by allowing us to access the digital space, but we must never forget that they cannot take responsibility: making decisions is a fuzzy area, where logic is not enough and ethics, social interactions and cultural environments shape and justify our actions and decisions.

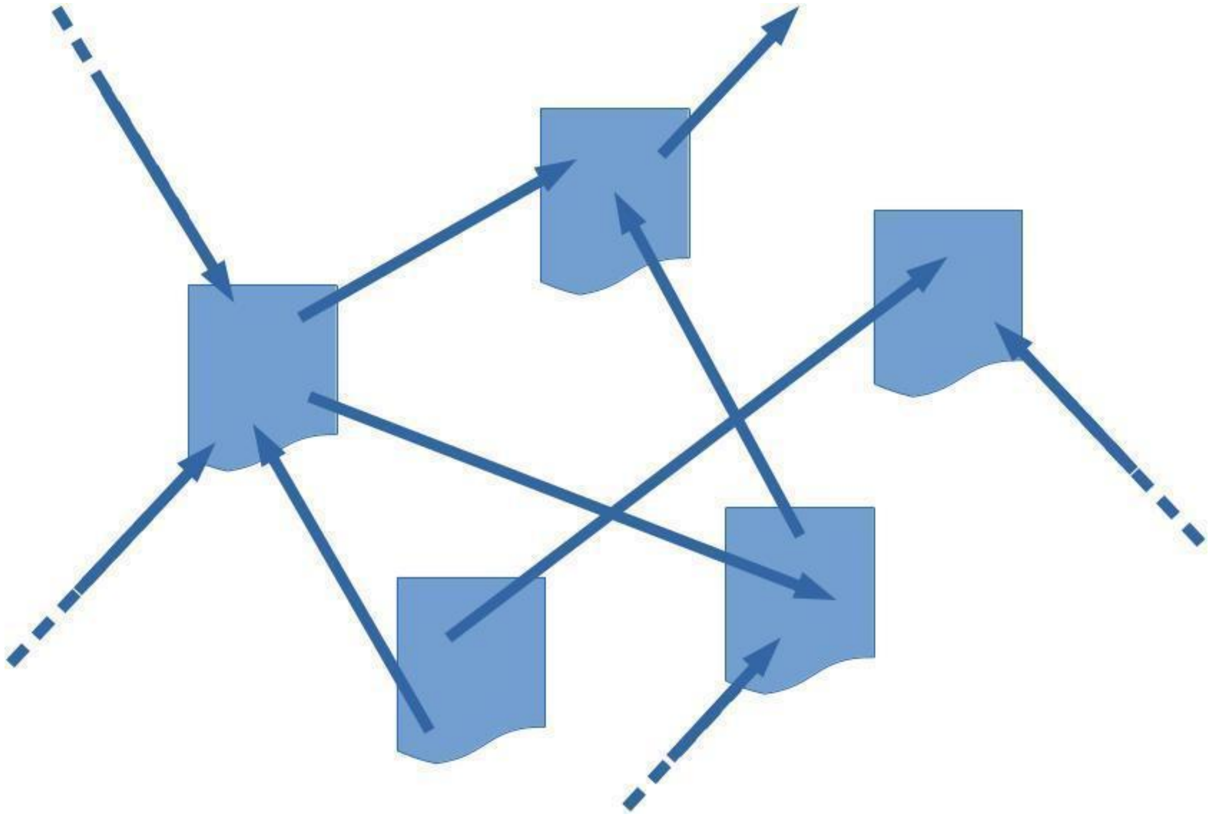

Figure 1: Objects on the internet are organised in networks. For example, web pages contain hyperlinks that connect them to other web pages, creating a network structure. This network allows for a "navigable" consumption of data, news, and information. Users can click on these links to move from one web page's content to another, effectively navigating the digital space.

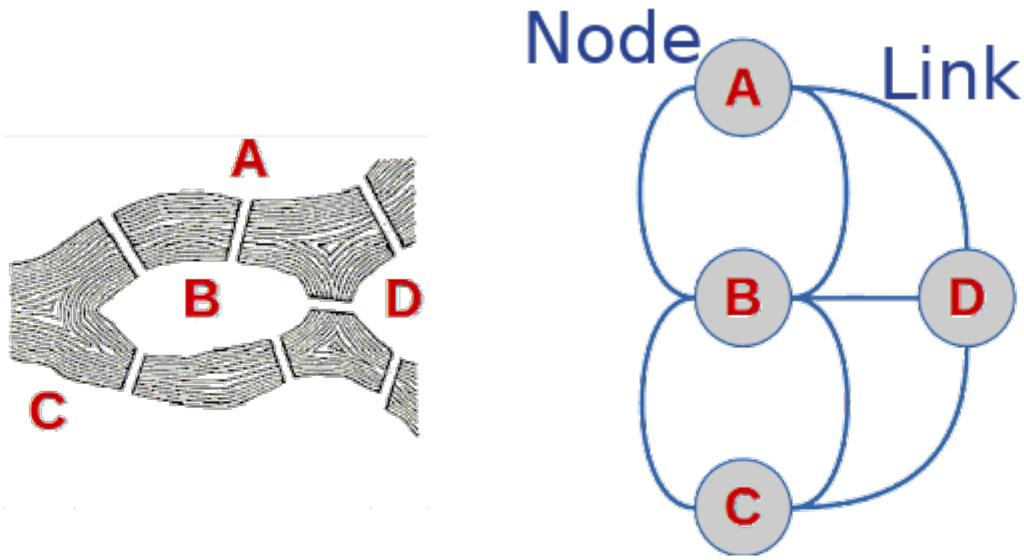

Figure 2: In 1736, the mathematician Leonhard Euler tackled this problem: «In Königsberg, Prussia, there is an island called Kneiphof, and the river that surrounds it is divided into two branches, as can be seen in the figure; the branches of this river are equipped with seven bridges. Regarding these bridges, it was wondered whether it was possible to build a path in such a way as to pass through each bridge once and only once. And I was told that some denied it and others doubted that this could be done, but no one was certain. From this, I have drawn this general problem: whatever the configuration and distribution of the river branches and whatever the number of bridges, it is possible to discover whether it is possible to pass through each bridge once and only once?» By simplifying the areas of the city as nodes (A, B, C, and D) and the bridges that connected them as links, Euler not only found the solution but also founded the modern theory of networks.

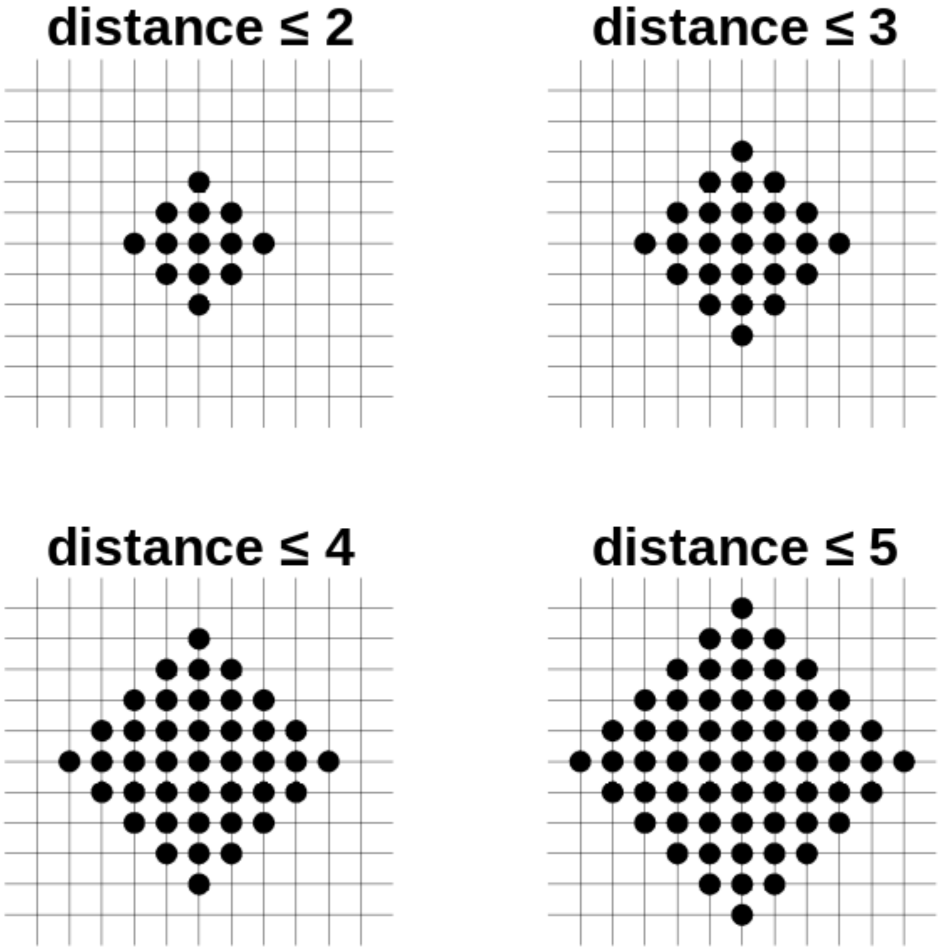

Figure 3: Comparing Navigational Efficiency: If the World Wide Web (WWW) were organized like the streets of Manhattan, accessing nearby pages would require significantly fewer clicks, but reaching distant ones could prove to be a daunting task. In the picture: the square grid of black lines (aka a "Manhattan grid"), where points reachable within a specified distance (i.e. number of "hops") are represented by filled circles. Understanding the hyperbolic nature of digital spaces sheds light on the challenges of digital exploration.

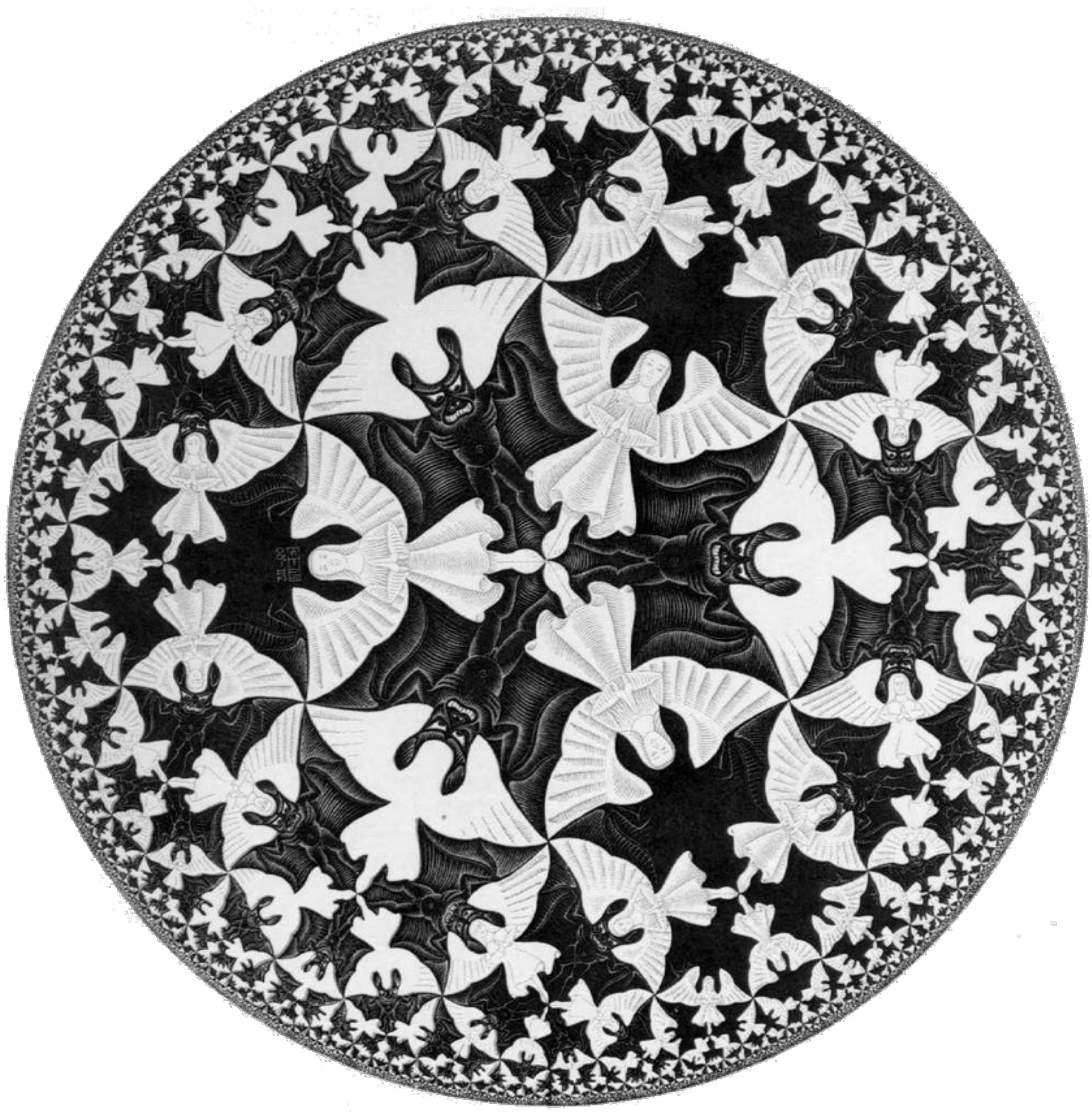

Figure 4: Artistic representation of the neighbourhood of a hyperbolic space (M.C. Escher's "Circle Limit IV", ©1997 Cordon Art Baarn Holland, All rights reserved). The centre of the disk represents the origin of our exploration; moving away from the centre, we encounter an exponentially growing number of neighbours (white angels and black demons). This artistic representation conveys the idea of how a hyperbolic space differs from our intuitive understanding of space – distances between objects rapidly shrink as one moves away from the centre, leading to a unique and intricate geometric structure.

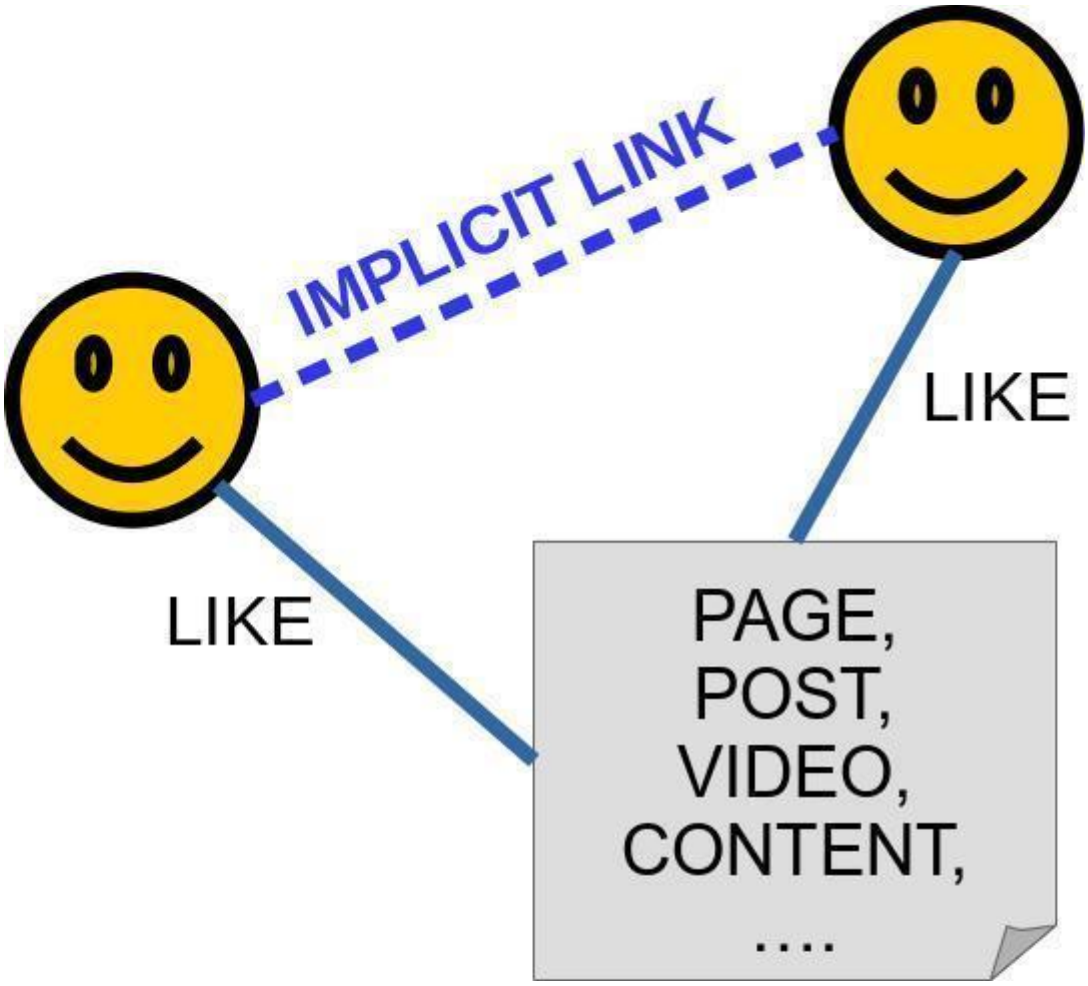

Figure 5: Example of an *implicit link in digital spaces*. Implicit links are not based on direct connections or explicit relationships between users but rather on inferred connections derived from shared characteristics, behaviours, or preferences. In this context, algorithms and data analysis play a crucial role in identifying and establishing these implicit connections and in building implicit networks that group together users with similar characteristics or interests.